A POSSIBLE CASE IN THE THEORETICAL DESCRIPTION OF THE RELATION BETWEEN THE STRESSES AND DEFORMATIONS OF AN ELASTOPLASTIC-CREEP MATERIAL

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UDC 539.37

In constructing the differential equation for stresses and deformations and their rates, it is proposed to use the nonlinear stress-deformation equations in the form of a quadratic parabola, when the loadings are instantaneous or very slow. Relaxation curves and curves of the creep and rate of creep of an elastoplastic material with nonlinear creep are obtained.

The differential equations for the stresses and deformations for materials with creep properties are usually assumed to be linear when the loading is instantaneous or very slow. Thus, for a Kelvin body

$$\sigma + \tau\dot{\sigma} = H\varepsilon + \tau E\dot{\varepsilon} \tag{1}$$

Assuming the $\sigma-\varepsilon$ relation to be nonlinear for instantaneous or very protracted loading, we can express the changes in the moduli of elasticity as

$$E' = E(1 - a\varepsilon), \qquad H' = H(1 - b\varepsilon)$$

Then Eq. (1) can be written as

$$\sigma + \tau\dot{\sigma} = H\varepsilon(1 - b\varepsilon) + \tau E\dot{\varepsilon}(1 - a\varepsilon) \tag{2}$$

When loading is very rapid, we can neglect the stresses and deformations σ and ε by comparison with their rates $\dot{\sigma}$, $\dot{\varepsilon}$, and Eq. (2) becomes

$$\dot{\sigma} = E\dot{\varepsilon}(1 - \alpha\varepsilon)$$

The term ε in the parentheses cannot be ignored by comparison with unity. At a constant loading rate $\dot{\sigma} = v$,

$$v = E\dot{\varepsilon}(1 - a\varepsilon)$$

Integrating, we have

$$vt = E\varepsilon(1 - {}^1/_2\, a\varepsilon), \quad \text{or} \quad \sigma = E\varepsilon(1 - {}^1/_2\, a\varepsilon)$$

This is the equation for a parabola. Denoting the coordinates of the vertex by σ_* (the greatest possible stress, or "yield point") and $\varepsilon_* = 1/a$ (the deformation when the yield point is reached, or the deformation at which failure occurs), we obtain an equation for the stresses and deformations when loading is instantaneous in the form

$$\sigma = E\varepsilon(1 - {}^1/_2\varepsilon / \varepsilon_*) \tag{3}$$

When loading is very slow, the rates $\dot{\sigma}$, $\dot{\varepsilon}$ in (2) can be ignored in comparison with σ, ε. Denoting the coordinates of the vertex of the parabola in this case by σ^*, $\varepsilon^* = b/2$, we have

$$\sigma = H\varepsilon(1 - {}^1/_2\,\varepsilon / \varepsilon^*) \tag{4}$$

Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 145-147, July-August, 1971. Original article submitted January 14, 1971.

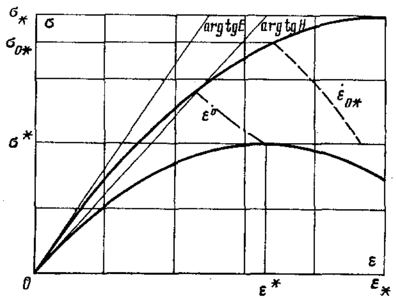

Fig. 1

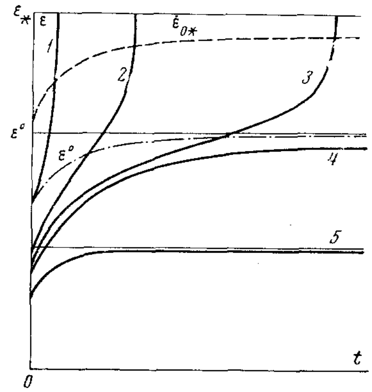

Fig. 2

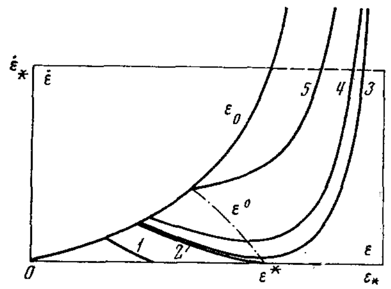

Fig. 3

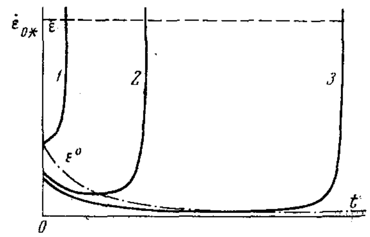

Fig. 4

Equations (3) and (4) are shown in Fig. 1 for the case where $\varepsilon^* < \varepsilon_*$. In general, we can have $\varepsilon^* \geq \varepsilon_*$ also, and even $\sigma^* > \sigma_*$, but in all cases we must have $E \geq H$, which follows from the essential nature of the concepts of instantaneous and protracted moduli.

The differential equation (2) finally has the form

$$\sigma + \tau\dot{\sigma} = H\varepsilon\left(1 - \frac{\varepsilon}{2\varepsilon^*}\right) + \tau E\dot{\varepsilon}\left(1 - \frac{\varepsilon}{\varepsilon_*}\right) \tag{5}$$

Let us apply an instantaneous stress σ_0. Then, from (3), the deformation is

$$\varepsilon_0 = \varepsilon_*(1 - \sqrt{1 - \sigma_0/\sigma_*}) = 2\sigma_* E^{-1}(1 - \sqrt{1 - \sigma_0/\sigma_*})$$

This deformation has elastic $\varepsilon^e = \sigma_0/E$ and plastic $\varepsilon^p = \varepsilon_0 - \varepsilon^e$ components.

If we now keep the deformation ε_0 constant, then $\dot{\varepsilon} = 0$, and from (5) we have the equation

$$\sigma + \tau\dot{\sigma} = H\varepsilon_0(1 - \varepsilon_0/2\varepsilon^*)$$

Solving it, we obtain the relaxation equation

$$\sigma = H\varepsilon_0\left(1 - \frac{\varepsilon_0}{2\varepsilon^*}\right) + \left[E\varepsilon_0\left(1 - \frac{\varepsilon_0}{2\varepsilon_*}\right) - H\varepsilon_0\left(1 - \frac{\varepsilon_0}{2\varepsilon^*}\right)\right]e^{-t/\tau}$$

This is the usual relaxation equation for a Kelvin body [1, 2].

If we keep the stress σ_0 constant, then $\dot{\sigma} = 0$, and from (5) we have

$$\sigma_0 = H\varepsilon\left(1 - \frac{\varepsilon}{2\varepsilon^*}\right) + \tau E\dot{\varepsilon}\left(1 - \frac{\varepsilon}{\varepsilon_*}\right) \tag{6}$$

The solution of this equation has the following form:

For $\sigma_0 > \sigma^*$

$$t = \frac{\tau E\varepsilon^*}{H\varepsilon_*}\left[\ln\frac{\theta(\varepsilon_0)}{\theta(\varepsilon)} + \frac{2\eta(\varepsilon_*)}{\vartheta}\left(\operatorname{arc\,tg}\frac{\eta(\varepsilon)}{\vartheta} - \operatorname{arc\,tg}\frac{\eta(\varepsilon_0)}{\vartheta}\right)\right]$$

For $\sigma_0 < \sigma^*$

$$t = \frac{\tau E \varepsilon^*}{H \varepsilon_*}\left[\ln\frac{\theta(\varepsilon_0)}{\theta(\varepsilon)} - \frac{2\eta(\varepsilon_*)}{i\vartheta}\left(\operatorname{Arth}\frac{\eta(\varepsilon)}{i\vartheta} - \operatorname{Arth}\frac{\eta(\varepsilon_0)}{i\vartheta}\right)\right]$$

$$\theta(\varepsilon) = \left(\frac{\varepsilon}{\varepsilon^*}\right)^2 - \frac{2\varepsilon}{\varepsilon^*} + \frac{\sigma_0}{\sigma^*}, \quad \eta(\varepsilon) = \frac{\varepsilon}{\varepsilon^*} - 1, \quad \vartheta = \sqrt{\sigma_0/\sigma^* - 1}$$

For $\sigma_0 = \sigma^*$

$$t = \frac{2\tau E \varepsilon^*}{H \varepsilon_*}\left(\frac{\varepsilon_* - \varepsilon^*}{\varepsilon^* - \varepsilon} - \frac{\varepsilon_* - \varepsilon^*}{\varepsilon^* - \varepsilon_0} + \ln\frac{\varepsilon^* - \varepsilon_0}{\varepsilon^* - \varepsilon}\right)$$

The creep curves are shown in Fig. 2 (for ε_*, ε_*, σ_*, σ^* taken from Fig. 1).

The curves 1, 2, 3, 4, 5 correspond to $\sigma_0 = 1.4\ \sigma^*$, $1.1\ \sigma^*$, $1.02\ \sigma^*$, σ^*, $0.75\ \sigma^*$.

For $\sigma_0 \leq \sigma^*$ the deformations tend asymptotically to the following value, defined by (4), as $t \to \infty$:

$$\varepsilon_\infty = \varepsilon^*\left(1 - \sqrt{1 - \frac{\sigma_0}{\sigma^*}}\right) = \frac{2\sigma^*}{H}\left(1 - \sqrt{1 - \frac{\sigma_0}{\sigma^*}}\right)$$

Failure does not occur. When $\sigma_0 > \sigma^*$, after a longer or shorter time interval the deformations reach the value ε_*, and the material fails. In accordance with this we can identify σ^* with the concept of the long-term (protracted) resistance limit.

In all cases when $\sigma_0 > \sigma^*$ the moment of failure is when the deformation reaches the value ε_*. However, as shown later, the moment of failure can be analyzed differently.

The rate of creep deformation is defined from (6) as

$$\dot{\varepsilon} = \frac{H \varepsilon_* \varepsilon^*}{2\tau E}\frac{\theta(\varepsilon)}{\varepsilon_* - \varepsilon} \tag{7}$$

Graphs of the change in the rate of deformation are given in Fig. 3. Curves 1, 2, 3, 4, 5 correspond to $\sigma_0 = 0.75\ \sigma^*$, σ^*, $1.02\ \sigma^*$, $1.1\ \sigma^*$, $1.4\ \sigma^*$. When $\sigma_0 \leq \sigma^*$, the rate of deformation falls from its initial value $\dot{\varepsilon}_0$ to zero when $\varepsilon = \varepsilon_\infty$.

When $\sigma_0 > \sigma^*$, the rate of deformation first decreases and then, when some critical deformation ε^0 is reached, begins to increase. The critical deformation can be defined from the minimum of Eq. (7):

$$\frac{\varepsilon^\circ}{\varepsilon_*} = 1 - \sqrt{1 + \frac{\sigma_0}{\sigma^*}\left(\frac{\varepsilon^*}{\varepsilon_*}\right)^2 - \frac{2\varepsilon^*}{\varepsilon_*}}$$

The curve joining the critical values ε° is shown in Figs. 1, 2, and 3 by a dash-dotted line.

For stresses from σ_0, corresponding to where ε° and ε_0 coincide, onwards, the deformation increases continuously from the onset of creep to the point of failure. At the moment of failure, when the limiting deformation ε_* is reached, the rate of deformation tends to infinity.

Since in the material tests the rate of deformation is not infinite at the moment of failure, but has a very large, though finite, value, after the moment of failure we can assume that the rate of deformation reaches a particular large value $\dot{\varepsilon}_*$. The "yield point" is not σ^*, but the stress $\sigma_{0*} < \sigma_*$ corresponding to $\dot{\varepsilon}_*$ (Fig. 1). The limiting deformation, the deformation at failure, is assumed to be ε_{0*}, corresponding to the assumed limiting rate and depending on the initial stress σ_0. The curve of σ_{0*} against ε_{0*} for some large rate of deformation $\dot{\varepsilon}_{0*}$ is shown in Fig. 1 by a dashed line.

We can also assume that the yield point is the point at which the rate of deformation $\dot{\varepsilon}_0$ starts to increase [2].

The creep and the rate of deformation curves obtained above (Figs. 2 and 3) do not contain segments where creep is established or the rate is constant. But the curvature of the creep lines when σ_0 is just greater than σ^* in the region near the point of inflection of ε° is very small and the deviation from a linear relation is not great. The graph giving the rate of deformation as a function of the time (Fig. 4) shows (curves 1, 2, 3 correspond to $\sigma_0 = 1.4\ \sigma^*$, $1.1\ \sigma^*$, $1.02\ \sigma^*$), that when σ_0 is slightly larger than σ^*, the rate of deformation is virtually constant over a long time.

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